Total No. of Questions—**12**] [Total No. of Printed Pages—**9**

Seat No.	

[5057]-51

S.E. (Electrical/Inst./Comp./I.T.) (First Semester)

EXAMINATION, 2016

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) In Section I, solve Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6.**

In Section II, solve Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Answers to the two sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Assume suitable data, if necessary.

(*v*) Neat diagrams must be drawn wherever necessary.

(*vi*) Use of non-programable electronic pocket calculator is allowed.

SECTION I

1. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^2 - 5D + 6)\, y = e^{4x} + 2$

(*ii*) $(D^2 + 2D + 1)\, y = e^{-x} x^3$

(*iii*) $(D^2 + 4)\, y = \sec 2x$ (by method of variation of parameters)

(*iv*) $x^2 \dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^2.$

(*b*) Solve the system of equations : [5]

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x + y = 0.$$

Or

2. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^2 - 4D + 4)\, y = \sin 2x$

(*ii*) $(D^2 - 1)\, y = x^3$

(*iii*) $\dfrac{dx}{y^2 z} = \dfrac{dy}{x^2 z} = \dfrac{dz}{y^2 x}$

(*iv*) $(x + 1)^2 \dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} + y = 4 \cos [\log (1 + x)].$

(*b*) An electric circuit consist of an inductance of 0.5 henry, a resistance R of 6 ohms and a condenser of capacitance 0.02 farad. If the differential equation of electric circuit is :

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0,$$

then find the charge q and current i at any time t, given that at $t = 0$, $q = 0.05$ coulomb, $i = \dfrac{dq}{dt} = 0$ when $t = 0$. [5]

3. (*a*) If

$$u = 3x^2 - 3y^2 + 2y,$$

find v such that

$$f(z) = u + iv$$

is analytic. Determine $f(z)$ in terms of z. [6]

(*b*) Evaluate : [5]

$$\int_C \frac{2z^2 + z + 5}{(z-1)^2} dz$$

where C is the circle $|z - 1| = 2$.

(*c*) Find the bilinear transformation which maps the points 1, i, $2i$ from z-plane into the points $-2i$, 0, 1 of w-plane respectively. [5]

Or

4. (*a*) Show that analytic function $f(z)$ with constant modulus is constant. [5]

(*b*) Evaluate by using Residue theorem :

$$\oint_C \frac{z^3 - 5}{(z+1)(z-2)} dz,$$

where C is the circle $|z| = 3$. [6]

(*c*) Show that the map

$$w = \frac{2z + 3}{z - 4}$$

transforms the circle

$$x^2 + y^2 - 4x = 0$$

into the straight line $4u + 3 = 0$. [5]

5. (*a*) Find the Fourier cosine integral representation of the function : [5]

$$f(x) = x, \quad 0 < x < a$$
$$= 0, \quad x > a.$$

(*b*) Solve the integral equation : [6]

$$\int_0^\infty f(x) \sin \lambda x \, dx = \begin{cases} 1, & 0 \le \lambda < 1 \\ 2, & 1 \le \lambda < 2 \\ 0, & \lambda \ge 2 \end{cases}.$$

(*c*) Find the *z*-transform of the following (any *two*) : [6]

(*i*) $f(k) = \cos(2k + 3), \ k \ge 0$

(*ii*) $f(k) = ke^{-ak}, \ k \ge 0$

(*iii*) $f(k) = \left(\frac{1}{2}\right)^{|k|}$, for all *k*.

Or

6. (*a*) Find inverse *z*-transform of the following (any *two*) : [6]

(*i*) $F(z) = \frac{z^2}{(z-2)(z-3)}$, if $2 < |z| < 3$

(*ii*) $F(z) = \frac{z}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}$ (by inversion integral method)

(*iii*) $F(z) = \frac{z}{(z-1)(z-2)}$, if $|z| \ge 2$.

(*b*) Solve the difference equation : [5]

$$f(k+1) + \frac{1}{2}f(k) = \left(\frac{1}{2}\right)^k, \; k \geq 0, \; f(0) = 0.$$

(*c*) Find the Fourier sine and cosine transform of the function

$$f(x) = e^{-x}$$

and hence show that :

$$\int_0^\infty \frac{\cos mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}.$$ [6]

SECTION II

7. (*a*) The first four moments of a distribution about the value 25 are –1.1, 89, –110 and 23300. From the given information obtain the first four central moments, coefficient of skewness and kurtosis. [8]

(*b*) Obtain regression line of *y* on *x* from the following data : [9]

x	*y*
2	11
4	8
6	9
8	7
10	5

Or

8. (*a*) On an average a box containing 10 articles is likely to have 2 defectives. If there is a consignment of 100 boxes, how many of them are expected to have three or less defectives ? [6]

(*b*) Suppose height of a student follows normal distribution with mean 190 cm and variance 80 cm^2. In a school of 100 students how many would you expect to be above 200 cm tall.

(Given : area corresponding to 1.118 is 0.3687) [5]

(*c*) Number of road accidents on a highway during a month follows a Poisson distribution with mean 5. Find the probability that in a certain month number of accidents on highway will be :

(*i*) atmost 2

(*ii*) at least 4. [6]

9. (*a*) A curve is given by the equations

$$x = t^2 + 1,$$

$$y = 4t - 3,$$

$$z = 2t^2 - 6t.$$

Find the angle between tangents at $t = 1$ and at $t = 2$. [5]

(*b*) Find the directional derivative of

$$\phi = xy^2 + yz^3$$

at (1, –1, 1) along the vector

$$\bar{i} + 2\bar{j} + 2\bar{k}.$$ [5]

(*c*) Solve any *two* : [6]

(*i*) Show that :

$$\nabla^2 f(x) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}.$$

(*ii*) Show that :

$$\nabla \cdot \left(\frac{\overline{a} \times \overline{r}}{r}\right) = 0.$$

(*iii*) If $\overline{u}$ and $\overline{v}$ are irrotational vectors, then prove that $\overline{u} \times \overline{v}$ is solenoidal vector.

Or

10. (*a*) Show that

$$\overline{F} = \left(2xy + z^3\right)\overline{i} + x^2\overline{j} + 3xz^2\overline{k}$$

is irrotational. Find scalar ϕ such that $\overline{F} = \nabla\phi$. [5]

(*b*) Find the directional derivative of

$$\phi = x^2yz^2$$

at (1, 1, –1) along tangent to the curve $x = -t$, $y = t^2$, $z = t$ at $t = 1$. [5]

(*c*) Solve any *two* : [6]

(*i*) Show that :

$$\nabla \cdot \left[r \nabla \frac{1}{r}\right] = -\frac{1}{r^2}.$$

(*ii*) Show that :

$$\nabla^2[r^2 \log r] = 6 \log r + 5.$$

(*iii*) If ϕ, ψ satisfy Laplace equation, then prove that the vector $(\phi\nabla\psi - \psi\nabla\phi)$ is solenoidal.

11. (*a*) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r} \quad \text{for} \quad \overline{F} = 3x^2\overline{i} + (2xz - y)\overline{j} + z\overline{k}$$

along a straight line

$$x = 2t, \; y = t, \; z = 3t$$

from $t = 0$ to $t = 1$. [6]

(*b*) Show that :

$$\iiint_V \frac{dv}{r^2} = \iint_S \frac{\overline{r} \cdot \hat{n}}{r^2} \, dS$$

where S is the surface enclosing the volume V. [5]

(*c*) Evaluate :

$$\iint_S (\nabla \times \overline{F}) \cdot d\overline{S}$$

for

$$\overline{F} = y\overline{i} + z\overline{j} + x\overline{k}$$

where S is the surface of the paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$. [6]

Or

12. (*a*) Evaluate :

$$\oint_C x^3 dx + 4xdy$$

over the circle

$$x^2 + y^2 = 4$$

by using Green's theorem. [6]

(*b*) Show that :

$$\iint_S (\phi\nabla\psi - \psi\nabla\phi).d\overline{S} = \iiint_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right)dV$$

where S is the surface enclosing the volume V. [5]

(*c*) Evaluate :

$$\iint_S (\nabla \times \overline{F}).d\overline{S}$$

where

$$\overline{F} = \left(x^3 - y^3\right)\overline{i} - xyz\overline{j} + y^3\overline{k}$$

and S is the surface

$$x^2 + 4y^2 + z^2 - 2x = 4$$

above the plane $x = 0$. [6]